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Denotational Mathematical Explanations for English Grammar Rules

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ABSTRACT

The present paper is an attempt in the direction of explaining rules of conversion of sentences from direct to indirect speech of English Grammar by the applications of matrix algebra and fuzzy logic.

1. INTRODUCTION

The notion of a grammar is central to most work in computational linguistics and natural language processing. It can be defined as a set of rules that can be applied to words to generate sentences in a language. David Crystal (2008) defines grammar as the study of the way words, and their component parts, combine to form sentence. Dictionaries define grammar as the rules and explanations which deal with the forms and structure of words (morphology), their arrangement in phrases and sentences (syntax), and their classification based on their function (parts of speech). Different languages provide different grammatical means of speech reporting and of making the difference between direct and indirect speech. Direct quotations not only create rhetorical effect of vividness and immediacy but also establish interpersonal involvement while the Reported speech has attracted the attention of scholars in several diversified and interesting fields such as linguistics, poetics, logic and the philosophy of language. We can find detailed account of different speech acts and trends in linguistics in the book edited by Coulmas

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and Ehlich (1986). They have explored the structural and functional problems of integrands when another's speech is expressed in one's own. In the book entitled "Narrative Theory: Critical concepts in literary and cultural studies" edited by Mieke Bal (2004), direct and indirect speeches have been characterized through the addition of certain rules to the base. The logic of indirect speech has been discussed by Steven Pinker (2008) by proposing a three part of indirect speech based on the idea that human communication involves a mixture of cooperation and conflict.

The need for closer contact between linguists and mathematicians has been felt since the beginning of this century and the affinity between these two branches has deepened with time. The necessity of mathematical formalism of different grammatical aspects has increased in today's perspective as different languages follow different rules of grammar. When machine translation is performed, the machine must know the grammatical rules of the source language and the target language if the final translation is to be grammatically correct. Masud et al (2003) have developed a general algorithm for translation of one natural language into another with the restriction of fixing up the input and output natural languages. Lambek in (2004) &, (2008) has given computational algebraic approach to English grammar by exploring an algebraic approach based on the notion of a ''pregroup,'' a partially ordered monoid in which each element has both a left and a right ''adjoint.'' Y.Wang et al (2009) have realized the importance of new forms of applied mathematics collectively known as denotational mathematics for explanation of the abstract, rigorous, and expressive needs in cognitive informatics, intelligence science, software science, and knowledge science. They have also presented a formal syntax framework of natural languages for computational linguistics so that the abstract syntax of natural languages, particularly English, and their formal manipulations could be described.

Khulbe et al (2009) have made an attempt to get the right grammatical patterns in different subject classes and their corresponding verbs forms so that transformation process from one to another form may be performed. To achieve this object, they have first defined a Group structure, closure with the grammatically correct patterns and then have described homomorphism on this group to show how the transformation takes place in between any two

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tense forms. Khulbe et al, in another work have defined the transformation processes from active to passive voice of the English language with the help of group theory of algebra, system tools and computer scheduling logics like PERT and CPM. In another work,

Our concern is to get the right grammatical patterns in different subject classes and their corresponding verbs forms so that transformation process from one to another form may be performed. The history of all branches of knowledge shows that many branches of mathematics have been created in order to meet their abstract, rigorous, and expressive needs. The application of logic to grammar is a fundamental issue in philosophy and has been investigated by such renowned philosophers as Leibniz, Bolzano. We are making an attempt in the direction of unifying logic and mathematics in the form of fuzzy logic and matrix algebra so that conversion rules from direct to indirect narration could be explained mathematically. The present work is a step forward in the direction of mathematically explaining all English Grammar rules so that software of English Grammar could be finally generated for the benefit of English learners.

2. PRELIMINIRIES

Any sentence in direct speech contains speaker (P), addressee(Q), reporting verb(S) and the inverted commas(R) followed by reported speech(T). Symbolically we can express it as-

$$P \wedge S \wedge Q \wedge R \rightarrow T$$

The reported speech *T* can further be segmented into $\alpha + \beta + \gamma$, where

$\alpha \in N$, *N* being the set of nouns, pronouns, and possessives;

$\beta \in D$, where *D* is the set of verbs, auxiliaries, modals, words expressing time and place

$\gamma \in C$ is the set of words other than the elements of *N* and *D* that is *C=T-(NUD)*

Thus $P \wedge S \wedge Q \wedge R \rightarrow \alpha + \beta + \gamma$……. (2.1)

The set *N* further consists of persons of first, second and third type and so

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$N = \{U, V, W\}$, where

$U = [\alpha_{11} \quad \alpha_{12} \quad \alpha_{13} \quad \alpha_{14}]$ …………..(2.2),

the elements being

$\alpha_{11} = I/We$; $\alpha_{12} = Me/Us$; $\alpha_{13} = My/Our$; $\alpha_{14} = Mine/Ours$

$V = [\alpha_{21} \quad \alpha_{22} \quad \alpha_{23} \quad \alpha_{24}]$ ………………(2.3),

where

$\alpha_{21} = You\,(subject)$; $\alpha_{22} = You\,(object)$; $\alpha_{23} = Your$; $\alpha_{24} = Yours$

and $W = [\alpha_{31} \quad \alpha_{32} \quad \alpha_{33} \quad \alpha_{34}]$ ………..(2.4),

where

$$\left.\begin{array}{l}\alpha_{31} = he \backslash she \backslash they \backslash proper\,noun;\ \ \alpha_{32} = him \backslash her(pronoun) \backslash them\,; \\ \alpha_{33} = his \backslash her\,(adjective) \backslash their\ ;\ \ \alpha_{34} = theirs\end{array}\right\}$$

In general, we can write

$N = \{\alpha_{ij}\ ;\ i = 1,2,3\ ;\ j = 1,2,3,4\}$

$\alpha_{ij} \in U$, *V, W, if i=1 , 2 ,3 respectively*……(2.5)

The set D set of verbs, auxiliaries, modals, words expressing time and place can be defined as under-

$D = \{\beta_1, \beta_2, \beta_3, \ldots \beta_{23}\} s.t.$

β_1 = has/ have, β_2 = am/ is/are , β_3 = these, β_4 = this, β_5 = now, β_6 = here, β_7 = ago, β_8 = thus, β_9 = today, β_{10} = tomorrow, β_{11} = yesterday , β_{12} = come, β_{13} = shall , β_{14} = will, β_{15} = may , β_{16} = can, β_{17} = past form of all verbs , β_{18} = present form of all verbs or present form of all verbs with an affix 's' or 'es' as the case may be , β_{19} = past continuous form of all verbs, β_{20} = last, β_{21} = next, β_{22} = henceforward, β_{23} = tonight

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3. MATHEMATICAL FORMULATION

Whenever it is desired that a sentence be converted from direct to indirect narration then we can explain this process under three steps:

Step I

The transformation process from direct to indirect narration can be understood to take place in the form of following matrix multiplication on *P, Q, R* and *S* (defined in the preliminaries by expression (2.1))

$$[P \quad Q \quad R]\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & m \end{bmatrix}$$

$$= \left[I : P \to P \qquad I : Q \to Q \quad m : R \to R^{'}\right] = \left[P \qquad Q \quad R^{'}\right] \; \ldots\ldots(3.1)$$

or

$$MN = M^{*} \ldots\ldots\ldots(3.2)$$

In the above process R' stands for ''that''

The transformation of reporting verb can be formalized mathematically as: $\left[S \quad S*\right]\begin{bmatrix} I & 0 \\ 0 & m' \end{bmatrix}$

.........(3.3)

If the reported verb *S* is ''said'', *S* = said, *S** = *0*

If the reported verb is ''said to'', *S** = said to, *S* = 0

The transformation shall take place as

$I : S \to S$ and $m' : S* \to S**$, where $S**$ stands for "told".

Step II

Conversion on N

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Conversion of *N* on transformation from direct to indirect speech is governed by the following rule:

$$[U \quad V \quad W]\begin{bmatrix} L & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & W \end{bmatrix} = [f : U \to L \quad g : V \to M \quad I : W \to W] \text{............(3.4)}$$

$$\textbf{or} \quad X \; Y = Z \quad \text{..............(3.5)}$$

In the above expression matrix X represents the person of α (noun/pronoun in the reported speech), whose specification can be defined as:

If $\alpha \in U$, U becomes operative and $V = W = 0$ in *X*.

The matrix *Y* of expression (3.5) represents the person (*L*) of speaker (*P)* and the person (*M*) of the addressee (*Q*) and Matrix *Z* represents the corresponding transformations defined in (3.4).

The transformations from $U \to L$ *and* $V \to M$ can be carried out by the application of fuzzy logic on the multiplication of two matrices, the first one being [*E*] having elements of either U or V present in the reported speech and the second one [*F*] is generated by all the elements of U , V ,*W* in first, second and third column of (3.4),respectively.

In order to be more elaborative, we can consider multiplication of E and F as

$$[\alpha_{ij}]_{1\times 4} [\alpha_{ji}]_{4\times 3}$$

Such that for $\alpha_{ij} \in U$, the value of EF shall be

$$[\alpha_{11} \quad \alpha_{12} \quad \alpha_{13} \quad \alpha_{14}]\begin{bmatrix} \alpha_{11} & \alpha_{21} & \alpha_{31} \\ \alpha_{12} & \alpha_{22} & \alpha_{32} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} \\ \alpha_{14} & \alpha_{24} & \alpha_{34} \end{bmatrix} \text{.........(3.6)}$$

and for $\alpha_{ij} \in V$, its value shall be

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$$[\alpha_{21} \quad \alpha_{22} \quad \alpha_{23} \quad \alpha_{24}]\begin{bmatrix} \alpha_{11} & \alpha_{21} & \alpha_{31} \\ \alpha_{12} & \alpha_{22} & \alpha_{32} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} \\ \alpha_{14} & \alpha_{24} & \alpha_{34} \end{bmatrix} \ldots\ldots\ldots(3.7)$$

The degree of presence in the reported speech can be ascribed to the elements of E by fuzzy logic as under:.

If μ_E is the membership function of E then

$$\mu_E = \begin{Bmatrix} \alpha_{ij} = 1 \, if \alpha_{ij} \in T \, (reported\ speech) \\ \alpha_{ij} = 0 \, if \alpha_{ij} \notin T \, (reported\ speech) \end{Bmatrix}$$

e.g., If $\alpha_{11} \in T\,(reported\ speech) \Rightarrow \alpha_{11} = 1 \; ; \alpha_{12}, \alpha_{13}, \alpha_{14}$=0 in E.

The degree of belongingness can be ascribed owing to the membership function μ_U , μ_V , μ_W of the following three sets, namely U,V,W as:

If $f: U \rightarrow U \; or \; g: V \rightarrow U$ then $\mu_U = \begin{Bmatrix} \alpha_{ij} = 0 \;\; if \;\; j \neq 1 \\ \alpha_{ij} = \dfrac{j}{i+j} if \;\; j = 1 \end{Bmatrix}$…….(3.8)

If $f: U \rightarrow V \; or \; g: V \rightarrow V$ then $\mu v = \begin{Bmatrix} \alpha_{ij} = 0 \;\; if \;\; j \neq 2 \\ \alpha_{ij} = \dfrac{j}{i+j} if \;\; j = 2 \end{Bmatrix}$……..(3.9)

If $f: U \rightarrow W \; or \; g: V \rightarrow W$ then $\mu_w = \begin{Bmatrix} \alpha_{ij} = 0 \;\; if \;\; j \neq 3 \\ \alpha_{ij} = \dfrac{j}{i+j} if \;\; j = 3 \end{Bmatrix}$…….(3.10)

We have represented the matrix of the membership function μ of the elements of F by F^* **.** When we considering one element at a time in E, then the element corresponding to the non zero entry in the product will be the image of α_{ij} in the indirect speech.

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Step III

Conversion on D

Representing B, A, C' as present, past and future forms of verbs respectively and assigning them the values $m = 0, 1, 2$, we can define the degree of tenses μ_T as follows:

$$\mu_t = \left\{\frac{m}{3} : m = 0,1,2\right\}......................(3.11)$$

$$such\ that\ T = \left\{0, \frac{1}{3}, \frac{2}{3}\right\}.................(3.12)$$

Now let ξ be the set of transformations for conversion of D from direct to indirect form, defined as

$$\xi = \begin{cases} h : D \to R^* if\ \mu_r = 0 \\ I : D \to D if\ \ \mu_r > 0 \end{cases} \qquad ....(3.13),$$

where the set R^* is defined as follows:

$R^* = \{\gamma_1, \gamma_2, \gamma_3, ..., \gamma_{23}\}$, where

γ_1 = had, γ_2 = was/were, γ_3 = those, γ_4 = that, γ_5 = then , γ_6 = there , γ_7 = before , γ_8 = so , γ_9 = that day , γ_{10} = the next day , γ_{11} = the day before , γ_{12} = go , γ_{13} = should , γ_{14} = would , γ_{15} = might , γ_{16} = could , γ_{17} = past perfect form of all verbs , γ_{18} = past form of all verbs , γ_{19} = past perfect continuous form of all verbs , γ_{20} = previous , γ_{21} = the following , γ_{22} = thenceforward , γ_{23} = that night

The transformations mentioned in expression (3.13) are defined as:

from $D \to R$ is it is h $(\beta_i) = \gamma_i$..............(3.14)

and for I it is $I(\beta_i) = \beta_i$............................(3.15)

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4. CONVERSION OF AFFIRMATIVE, NEGATIVE, IMPERATIVE, AND INTERROGATIVE SENTENCES.

(A) CONVERSION OF AFFIRMATIVE AND NEGATIVE SENTENCES

The conversion process shall take place immediately by the application of techniques discussed so far, as illustrated in the following example:

Illustrative Example I:

Direct speech: The teacher said to me, "You have not done your work well."

Structure of sentence is $P \Lambda Q \Lambda S \Lambda R \rightarrow \alpha + \beta + \gamma$ .... *from* (2.1),

where P = The teacher , Q = me , S= said , R = " "

Mathematical formulation of P, Q *and* R by result (3.1) shall be

$$[\text{The teacher} \quad \text{me} \quad \text{“ ”}] \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & m \\ & & \end{bmatrix}$$

= [The teacher me that] **…………**(4.1)

The conversion of S shall take place by result (3.3) as:

$$[0 \quad said \;\; to] \begin{bmatrix} I & 0 \\ 0 & m' \end{bmatrix} = [0 \quad told]$$

Hence said to is converted to **told.**

Mathematical formulation of N by (2.2), (2.3), (3.4) and (3.8) shall be

$$\alpha = I + my = \alpha_{11} + \alpha_{13} \; ; \; \alpha_{11} \; , \; \alpha_{13} \in U$$

Since Speaker P = 'The teacher' is the third person, therefore by result (2.4)

$P \in W$ *or* $L = W$;

and Q = ' me' is the first person, which implies $Q \in U$ *or* $M = U$ from (2.2).

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Also $\alpha = You + Your = \alpha_{21} + \alpha_{23}$ *where* $\alpha_{21}, \alpha_{23} \in V$ from (2.3)

So from (3.4)

$$[0 \quad V \quad 0]\begin{bmatrix} W & 0 & 0 \\ 0 & U & 0 \\ 0 & 0 & W \end{bmatrix} = [0 \quad g: V \rightarrow U \quad 0]$$

The explanation of the transformation *g:* $V \rightarrow U$ of the above result can be described as under:

Conversion of α_{21} = you is carried out with the help of result (3.8) as follows:

$$E=[1 \quad 0 \quad 0 \quad 0] \;; F^* = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{2}{3} & 0 & 0 \\ \frac{3}{4} & 0 & 0 \\ \frac{4}{5} & 0 & 0 \end{bmatrix}$$

and $EF^* = \left[\max(\frac{1}{2},0,0,0) \quad \max(0,0,0,0) \quad \max(0,0,0,0)\right] = \left[\frac{1}{2} \quad 0 \quad 0\right] = \alpha_{11}$ = I

Conversion of α_{23} = your, is carried out again with the help of result (3.8) as follows:

$$E=[0 \quad 0 \quad 1 \quad 0] \;; F^* = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{2}{3} & 0 & 0 \\ \frac{3}{4} & 0 & 0 \\ \frac{4}{5} & 0 & 0 \end{bmatrix}$$

and $EF^* = \left[\max(0,0,\frac{3}{4},0) \quad \max(0,0,0,0) \quad \max(0,0,0,0)\right] = \left[\frac{3}{4} \quad 0 \quad 0\right] = \alpha_{13}$ = my

as such ‘You’ shall be transformed as ‘I and ‘your’ as ‘my’.

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Mathematical formulation for D shall be done with the help of results (3.11) to (3.15) as under:

Here S = said $\in B$, thus $m = 0$

$$\Rightarrow \mu_m = \frac{0}{3} = 0.$$

$$\therefore \mu_m = 0 \Rightarrow transformation\ will\ be\ h : D \rightarrow R^*$$

$\beta = have = \beta_1$

$h(\beta_1) = \gamma_1 =$ had

Thus **have** is changed to **had** on conversion to indirect speech.

Hence the transformed sentence in Indirect speech is

The teacher **said to me** that **I had** not done **my** work well.

<u>EXCEPTION</u>

If "T ''represents some universal truth or habitual fact then D→D and N→N e.g.,

He says, " The moon revolves around the earth ."

He says that the moon revolves around the earth

REFERENCES

Coulmas Florian (ed.) (1986) Direct and Indirect speech (Trends in linguistics), Walter de Gruyter, Berlin, New York and Amsterdam.

David Crystal (2008) A Dictionary of Linguistics and Phonetics, 6th edition, Blackwell Publishing.

Khulbe Sumit , Sharma Richanshu and Dhami H.S.(2009) Mathematical Formalism and Computer Program for Tense Conversion in English Grammar International Transactions in Mathematical Sciences and Computer July-December 2009, Volume 2, No. 2, pp. 307-317 .

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Khulbe Sumit, Pande Rakesh and Dhami H.S.(2009) Mathematical formulation and Development of computer program for active to passive voice conversion, Accepted for publication in the Vol.1, No.3 issue of the Journal "International Transactions in applied Sciences".

Lambek J. (2004). A computational algebraic approach to English grammar, Syntax, 7(2), 128–147.

Lambek Joachim (2008). From word to sentence. A computational algebraic approach to Grammar, Polimetrica Publishers.

Masud M.A.N., Joarder M.M.M., Tariq-ul-Azam, Khulna M. (2003). A general approach to natural language conversion, Appeared in 7th International Multi Topic Conference, INMIC 2003.

Mieke Bal (ed.) (2004) Narrative Theory: Critical concepts in literary and cultural studies, Routledge, Taylor and Francis Group.

Pinker Steven, Nowak Martin A., and James J.Lee (2008) The logic of indirect speech, PNAS (Publication of the National Academy of Sciences of the USA, 105(3), 833-838.

YingxuWang, Du Zhang and Shusaku Tsumoto (2009) Preface: Cognitive Informatics, Cognitive Computing, and Their Denotational Mathematical Foundations (II), Fundamenta Informaticae 90 (2009) i–vii.